Colour-straight four-quark operators and lifetimes of beautiful hadrons

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Abstract

Using the relation between the harmonic oscillator wave function and the light quark scattering form factor, the expectation values of colour-straight four-quark operators are evaluated and found to be directly proportional to the cubic power of the oscillator strength. It is predicted that the ratio $\tau(\Lambda_b)/\tau(B) \approx 0.79(0.84)$ due to the factorizable (nonfactorisable) piece, against the experimental 0.79 ± 0.06. Notwithstanding the numerical prediction, the present study shows that the four-quark operators play a role as far the lifetimes of b-flavoured hadrons are concerned.

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I. INTRODUCTION

In the description of heavy hadron decays by heavy quark expansion (HQE), the preasymptotic effects appearing at next-to-leading order and beyond are vital in predicting the decay properties accurately. These effects are due to the operators of dimensions D = 5 and 6. At $1/m_Q^2$, the operators are suppressing the leading order. The evaluation of D = 5 operators which describe the motion of the heavy quark inside the hadron and the chromomagnetic interaction is definite. The estimation of the D = 6 operators which are four-quark operators (FQO) containing both heavy and light fields is based on the vacuum insertion assumption for mesons and on the quark models for baryons. Though their effects are negligible as the heavy quark's volume occupation is of the order of $(\Lambda_{QCD}/m_Q)^3$ but are considerably enhanced due to partial compensation by the four-quark phase-space, these operators are predicting the lifetime differences in the world of the hadrons of given flavour quantum number. Therefore, accurate value of the FQO is necessary due to the confrontation existing between theory and experiment over the hadronic properties like the experimentally smaller than theoretically expected lifetime of Λ_b and the theoretically smaller than experimentally predicted semileptonic branching ratio of B-meson. Theoretically upto order $(1/m_b{}^2)$ [1,2]

$$\frac{\tau(B^+)}{\tau(B^0)} = 1 + 0.05 \, O \left(\frac{f_B}{200 MeV} \right)^2$$
$$\frac{\tau(B_s)}{\tau(B^0)} = 1 + 0.01$$
$$\frac{\tau(\Lambda_b)}{\tau(B^0)} = 0.9 \tag{1}$$

whereas experimentally [3],

$$\frac{\tau(B^+)}{\tau(B^0)} = 1.04 \pm 0.04$$
$$\frac{\tau(B_s)}{\tau(B^0)} = 0.99 \pm 0.05$$
$$\frac{\tau(\Lambda_b)}{\tau(B^0)} = 0.79 \pm 0.06 \tag{2}$$

The agreement among the B mesons is as expected of but not so between the Λ_b and B. The later issue continues to be central point of the physics of heavy quark hadrons. It is suspected that the explanation for these discrepancies within the HQET framework is hidden in the yet-not-satisfactorily-understood FQO.

As regards the evaluation of the FQO, there were two works [4,5] which attempted to explain substantially the enhanced decay rate of Λ_b, whereas the work of P. Colangelo and F. De Fazio [6] which is QCD sum rules based prediction leads to conclude that the reason for the smaller lifetime is not due to FQO. In Ref. [4], the authors evaluated the FQO parameterising the matrix elements in terms of hadronic parameters which are not practically known. But the parameters have been calculated using QCD sum rules [7]. However, the prediction is not able to account for the lifetime difference between Λ_b baryon and B meson. On the other hand, the author of Ref. [5] used quark model and accounted for the FQO for 13% of the required enhancement in the Λ_b decay rate. The above estimation used yet to be confirmed result of DELPHI collaboration [8] on the mass splitting of Σ_b^* and Σ_b. The same method has been modified by taking the logarithmic dependence of the wave function at the origin and this explains the difference between the decay rates of B meson and Λ_b baryon to the extent of 40% [9]. Since the striking point in the evaluation of the FQO is not yet obtained to clear the situation in one way or the other, it is important and interesting to explore other avenues to estimate the four-quark matrix elements.

In this paper, we adopt the colour-straight formalism approach of [10] to evaluate the expectation values of the four-quark matrix elements. On the specific choice of the harmonic oscillator wavefunction model for the form factor and slightly different potentialr for meson and baryon, it is found that

$$\frac{\tau(B^+)}{\tau(B^0)} = 1.00(1.03)$$
$$\frac{\tau(B_s)}{\tau(B^0)} = 1.00(1.02)$$
$$\frac{\tau(\Lambda_b)}{\tau(B^0)} = 0.79(0.84) \tag{3}$$

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where the values given within brackets are due to non-factorisable part of the FQO. These values are in agreement with the data, eq. (2).

In a recent work [10], Pirjol and Uraltsev discussed the four-fermion operators on certain quantum mechanical basis. In the nonrelativistic quark theory, the wave function density and diquark density are related to the associated operator

$$(\bar{b}_i \Gamma_b b^i)(\bar{q}_j \Gamma_q q^j), \tag{4}$$

where the $\Gamma_{b,q}$ are arbitrary Dirac structures, through

$$\frac{1}{2M_B} < B|(\bar{b}b)(\bar{q}q)|B > = |\Psi(0)|^2 \tag{5}$$

$$\frac{1}{2M_{\Lambda_b}} < \Lambda_b|(\bar{b}b)(\bar{q}q)|\Lambda_b > = \int d^3y |\Psi(0,y)|^2 \tag{6}$$

for meson and baryon respectively. The operators in Eq. (4) are colour singlet. As colour flows freely for these operators, they are called colour-straight. The expectation values of the operators in Eq. (4) are related to the observable, the transition amplitude of the light quark scattering off the heavy quark. Thus the determination is based on the knowledge of the light quark scattering form factor.

The wave function at the origin, in momentum representation, is given by

$$\Psi(0) = \int \frac{d^3\mathbf{p}}{(2\pi^3)^3} \Psi(\mathbf{p}). \tag{7}$$

The transition amplitude is then the Fourier transform of the light quark density distribution:

$$F(\mathbf{q}) = \frac{1}{2M_H} < H_b(\mathbf{q})|\bar{q}q(0)|H_b(0) > = \int d^3\mathbf{x} \Psi^*(\mathbf{x})\Psi(\mathbf{x}) e^{i\mathbf{q}\mathbf{x}}. \tag{8}$$

Integrating over all $\mathbf{q}$ yields the expectation value:

$$\int \frac{d^3\mathbf{q}}{(2\pi)^3} F(\mathbf{q}) = |\Psi(0)|^2 = \frac{1}{2M_H} < H_b|\bar{b}b\bar{q}q(0)|H_b > \tag{9}$$

For any Dirac structure Γ, the light quark current density and the light quark transition amplitude are given by:

$$J_\Gamma(\mathbf{x}) = \bar{q}\Gamma q(\mathbf{x}); \quad A_\Gamma(\mathbf{q}) = \frac{1}{2M_H} < H_b(\mathbf{q})|J_\Gamma(0)|H_b(0) > \tag{10}$$

where the $J_\Gamma(0)$ is gauge invarinat operator and not required to be a bilinear. Thus, for spin-singlet operators, we have

$$< H(\mathbf{q})|\bar{b}b J_\Gamma(0)|H_b(0) >= \int \frac{d^3\mathbf{q}}{(2\pi)^3} < H_b(\mathbf{q})|J_\Gamma(0)|H_b(0) > \tag{11}$$

And, for spin-triplet operators, similarly

$$< H(\mathbf{q})|\bar{b}\sigma_\mathbf{k}b \bar{J}_\Gamma(0)|H_b(0) >= \int \frac{d^3\mathbf{q}}{(2\pi)^3} < S_k \, H_b(\mathbf{q})|J_\Gamma(0)|H_b(0) > \tag{12}$$

with S/2 being the b-quark spin operator and

$$|S_k \, H_b >= \int d^3\mathbf{x}\bar{b}\sigma_k b(\mathbf{x})| \, H_b > \tag{13}$$

Equations (11) and (12) represent the general structure of four-quark operators. The above operators are local as required for by the HQE in the sense that the light quark operators enter at the same point as the heavy b-quark operators. These relations, Eqs. (11) and (12), hold equally for different initial and final hadrons having different momenta smaller than m_b.

Equation (4) resolves into spin-singlet and spin-triplet operators for $\Gamma_b = 1$ and $\Gamma_b = \gamma\gamma_5(= \sigma)$ respectively. The light quark elastic scattering is described by the form factor $F(q^2)$. In Ref. [10], the exponential ansatz and the two pole anstaz are used for the form factor. Both of them lead to a determination which make no difference for meson and baryon. This cannot truly be the case to represent $|\Psi(0)|^2$, the measure of the expectation values of FQO, for a baryon and a meson.

In the next section, the choice of the representation, harmonic oscillator wave function for the form factor and the potentials are discussed. The evaluation of the expectation values of the factorisable part of the FQO is presented in Sec. III. Corresponding non0factorisable part is given in Sec. IV. Estimation of the lifetimes ratio and the conclusion are given in Sec. V and VI respectively.

II. HARMONIC OSCILLATOR WAVE FUNCTION MODEL FOR FORM FACTOR

As will be discussed in the following sections, the expectation values of the colour-straight operators are parameterised in terms of a single form factor for both B-meson and Λ_b baryon. The extraction of the form factor involves assumption of a function such that it satisfies the constraints on the form factor that $F(q^2 = 0)$ is equal to the corresponding charge of the hadron. Then the form factor has to be extrapolated into the region where $q^2 > 0$. We take the hadronic wave function of ISGW harmonic oscillator model [11] for the form factor.

The wave functions of ISGW model are the eigenfunctions of orbital angular momentum $L = 0$ satisfying the overlapping integral

$$I(\mathbf{q}) = \int r^2 dr \Psi_f^*(r) \Psi_i(r) j_0(\mathbf{q}r). \tag{14}$$

The overlapping integral can be equated to the form factor. Hence for diffrent initial and final hadrons

$$F(\mathbf{q}^2) = N^2 exp[-q^2/2(\beta_f^2 + \beta_i^2)] \tag{15}$$

where N is the normalisation constant given by $[2\beta_f\beta_i/(\beta_f^2 + \beta_i^2)]^{3/2}$ and β's are oscillator strengths. For same initial and final hadrons, the transition amplitude is

$$\int \frac{d^3\mathbf{q}}{(2\pi)^3} F(q^2) = \frac{\beta^3}{4\pi^{3/2}} = |\Psi(0)|^2 \tag{16}$$

From the above equation, which is the central point of dicussion of this paper, it is obvious that the transition amplitude and hence the expectation values of four fermion operators are proportional to the third power of the oscillator strength of the hadron.

The calculation of β's can be made using the QCD inspired potential. In the present calculation, we use the potential for B-meson containing the Coulomb, confining and a constant terms:

$$V(r) = \frac{a}{r} + br + c \tag{17}$$

For a = -0.508, b = 0.182 GeV^2 and c = -0.764 GeV, and for quark masses m_q = 0.3 GeV (treating $m_u = m_d$), m_s = 0.5 GeV and m_b = 4.8 GeV, using variational procedure with the wave function given in position space as,

$$\Psi(0)_{1s} = \frac{\beta^{3/2}}{\pi^{3/4}} e^{\beta^2 r^2/2} \tag{18}$$

β_{B_q} = 0.4 GeV and β_{B_s} = 0.44 GeV.

For Λ_b baryon, following the similar procedure but for the potential of the form

$$V(r) = \frac{1}{2}\left(\frac{a}{r} + br + \beta r^2 + c\right) \tag{19}$$

where the r^2 term is a harmonic oscillator term justifying the consideration that Λ_b be a two body system and the same wave function of Eq. (18), one gets β_{Λ_b} = 0.72 GeV and for β_{Ξ_b} = 0.76 GeV for the values of mass of the diquark system 0.6 and 0.8 GeV respectively. The large value for the $\beta_{|Lambda}$ is due to the presence of the O(r^2) term in the potential. Otherwise, the value of β_Λ is no different than that of B$_s$. These values are used in the subsequent calculations in this paper. A comment is in order on the choice of the same wave function for baryon as for meson: In the usual procedure, the ground state wave function for baryon is

$$\Psi_{ground} \approx e^{-\alpha^2(r_\lambda^2 + r_\rho^2)/2} \tag{20}$$

where r$_{\lambda,\rho}$ are the internal coordinates for three body system. Due to the idea of considering the Λ_b as a system containing the bound state of light quarks and a heavy quark, the separation of the two light quarks which make the bound state is treated negligibly. This allows then that the baryon is a system of two body. It is a reasonable approximation only. The difference between a meson and a baryon is essentially due to the value of the oscillator strength.

III. EXPECTATION VALUES OF THE COLOUR-STRAIGHT OPERATORS

We evaluate the expectation values of the colour-straight operators only for the vector and axial-vector currents. Nevertheless the other currents can also be studied in the same

fashion. Both the currents are possible for B-meson while axial currents vanish for Λ_b baryon due to the light degrees of freedom which constitute a spinless bound state.

Essentially there is no difference between the exponential ansatz and the harmonic oscillator wave function in representing the behaviour of the form factor but they differ while fixing the scale: in the former case, the hadronic scale of one GeV is used whereas in the latter the same has been fixed by solving the Schrodinger equation. The two pole anstaz is based on the well founded experimental values. Basically the use of the harmonic oscillator wave function of the constiuent quark model is an alternate picture but in the very same lines of the two ansatz.

Hereinafter the operators are referred to by the following notaion: for meson

$$< O^q_{V,A} >_H = < H|\bar{b}\Gamma_{V,A}b\bar{q}\Gamma_{V,A}q|H >; \qquad < T^q_{V,A} >_H = < H|\bar{b}\Gamma_{V,A}t^a b\bar{q}\Gamma_{V,A}t^a q|H > \qquad (21)$$

where $\Gamma_{V,A} = \gamma_\mu, \gamma_\mu\gamma_5$ and $t^a t^b = 1/2 - 1/2N_c$. In what follows, q stands for u and d quarks and s for strange quark. And $< O_{V,A} >, < T_{V,A} >$ will be respectively denoted as $\omega_{V,A}, \tau_{V,A}$. For baryon, $< O(T)_V >$ correspond to $\lambda_{1,2}$ respectively.

A. B-meson

The parametrisation of the matrix element of the colour-straight operators for vector current is

$$< B(q)|J_{\Gamma_V}|B(0) >= -v_\mu F_B(q^2) \qquad (22)$$

with the constraints $F_B(0) = 1$ for valence quark current and $F_B(0) = 0$ for sea quark current. The former is relevant for the b-meson composition of quarks $b\bar{q}$. Then the corresponding transition amplitude is

$$A^B_{\Gamma_V} =< B(q)|O^q_V|B(0) >= -v_\mu \int \frac{d^3q}{(2\pi)^3} F_B(q^2) \qquad (23)$$

Under isospin SU(2) symmetry,

$$< B^- |O_V^q| B^- >= -2.88 \times 10^{-3} \ GeV^3; \qquad < B^- |T_V^q| B^- >= -9.61 \times 10^{-4} GeV^3 \qquad (24)$$

For B$_s$, we have

$$< B_s |O_V^q| B_s >= -3.83 \times 10^{-3} \ GeV^3; \qquad < B_s |T_V^q| B_s >= -1.28 \times 10^{-3} \ GeV^3 \qquad (25)$$

If the case of violation of isospin symmetry, and SU(3) flavour symmetry, is considered, then there comes Cabibbo mixing of eigenstates for d and s quarks among themselves. That is, $d' = d cos\theta_c + s sin\theta_c$ and $s' = s cos\theta_c + d sin\theta_c$. This we do not consider here.

For axial current there are two form factors which are related to one another due to conservation of the axial current, $\partial_\mu J_{\mu 5} = 0$, in the chiral limit. By the Goldberger-Treiman relation [12] which equates axial charge form factor to the coupling g$_{B^* B \pi}$ at q$^2 = 0$, the operators involving axial-currents are estimated in terms a single form factor. Thus, given the value of the coupling g, the extraction of the transition amplitude is similar to the B-meson case.

Making use of

$$(S_b \epsilon)|B(q) >= |B^*(q, \epsilon) > \qquad (26)$$

and Eq.(12), the expectation values for the axial vector currents are given by

$$< B(q, \epsilon)^*| \sum_{q=u,d,s} \bar{q}\gamma_\mu \gamma_5 q |B(0) >= \epsilon_\mu^* G_1^{(0)}(q^2) - (\epsilon^* q) q_\mu G_0^{(0)}(q^2) \qquad (27)$$

$$< B(q, \epsilon)^*|\bar{q}\lambda^a \gamma_\mu \gamma_5 q |B(0) >= \{\epsilon_\mu^* G_1^{(0)}(q^2) - (\epsilon_\mu^* q) q_\mu G_0^{(0)}(q^2)\}\lambda_{ij}^q \qquad (28)$$

Finally the following equality leads the absence of the structure $(\epsilon^* q) v_\mu$

$$< B^*(q, \epsilon)|j_{\mu 5}(0)|B(0) >^*=< B(0)|j_{\mu 5}(0)|B^*(q, \epsilon) >=< B^*(0, \epsilon)|j_{\mu 5}(0)|B(q) > \qquad (29)$$

Following the Goldberger-Treiman relation, we have

$$G_1(q^2) = q^2 G_0(q^2) = g e^{-q^2/4\beta^2} \qquad (30)$$

Correspondingly, the expectation values ars

$$< B^-|O_A^q|B^- >= 8.63 \times 10^{-5} \ GeV^3; \qquad < B^-|T_A^q|B^- >= 2.88 \times 10^{-5} \ GeV^3 \qquad (31)$$

$$< B_s|O_A^s|B_s >= 1.15 \times 10^{-4} \ GeV^3; \qquad < B_s|T_A^s|B_s >= 3.84 \times 10^{-5} \ GeV^3 \qquad (32)$$

We have taken in the above esitmates the value g = - 0.03 [13,14].

B. Λ_b baryon

For Λ_b baryon, treating u and d quarks equally,

$$< \Lambda_b|O_V^q|\Lambda_b >= -1.69 \times 10^{-2} \ GeV^3; \qquad < \Lambda_b|T_V^q|\Lambda_b >= -5.64 \times 10^{-3} \ GeV^3 \qquad (33)$$

In the case of Ξ_b, we have,

$$< \Xi_b| \sum_{q'=u,d,s} O_V^{q'}|\Xi_b >= -2.01 \times 10^{-2} \ GeV^3; \qquad < \Xi_b| \sum_{q'=u,d,s} O_V^{q'}|\Xi_b >= -6.72 \times 10^{-3} \ GeV^3$$

$$(34)$$

There are corrections additionally to form factors due to charge radius. The same can be ignored as we are looking at the wave function density at the origin.

IV. NON-FACTORISABLE PART OF THE FQO

The nonfactorisable part of the FQO come in four. The following is one of the ways of parameterising them [4].

$$\frac{1}{2M_B} < B|(\bar{b}q)_{V-A}(\bar{q}b)_{V-A}|B >= \bar{f}_B^2 M_B B_1/2 \qquad (35)$$

$$\frac{1}{2M_B} < B|(\bar{b}t^a q)_{V-A}(\bar{q}t^a b)_{V-A}|B >= \bar{f}_B^2 M_B \epsilon_1/2 \qquad (36)$$

$$\frac{1}{2M_B} < B|(\bar{b}q)_{S-P}(\bar{q}b)_{S-P}|B >= \bar{f}_B^2 M_B B_2/2 \qquad (37)$$

$$\frac{1}{2M_B} < B|(\bar{b}t^a q)_{S-P}(\bar{q}t^a b)_{S-P}|B >= \bar{f}_B^2 M_B B_2/2 \qquad (38)$$

where $B_{1,2}$ and $\epsilon_{1,2}$ are hadronic parameters. They are related to $w_{V,A}$ and $\tau_{V,A}$ which are the expectation values of the operators $O_{V,A}$ and $T_{V,A}$ as defined earlier.

$$\bar{f}_B^2 M_B B_1 = \phi_1 = 4(\tau_V + \tau_A) + \frac{2}{N_C}(\omega_V + \omega_A) \tag{39}$$

$$\bar{f}_B^2 M_B B_2 = \phi_2 = -2(\tau_V - \tau_A) - \frac{1}{N_C}(\omega_V - \omega_A) \tag{40}$$

$$\bar{f}_B^2 M_B \epsilon_1 = \rho_1 = -\frac{2}{N_C}(\tau_V + \tau_A) + (1 - \frac{1}{N_C^2})(\omega_V + \omega_A) \tag{41}$$

$$\bar{f}_B^2 M_B \epsilon_2 = \rho_2 = \frac{1}{N_C}(\tau_V - \tau_A) - \frac{1}{2}(1 - \frac{1}{N_C^2})(\omega_V - \omega_A) \tag{42}$$

In the case the Λ_b baryon, the nonfactorisable piece corresponds to [4]

$$< (\bar{b}q)_{V-A}(\bar{q}b)_{V-A)} = -\frac{1}{2N_C}\lambda_1 - \lambda_2 \tag{43}$$

V. DECAY RATES AND LIFETIMES

The decay rates of the b-flavoured hadrons are given by

$$\Gamma(H_b \to H_c l \bar{\nu}_l) = \frac{G_f^2 m_b^5 |V_{cb}|^2}{192\pi^3} \left[(1 + \frac{\lambda_1 + 3\lambda_2}{2m_Q^2})f(x) - \frac{6\lambda_2}{2m_Q^2}f'(x) \right] \tag{44}$$

where, with $x = m_c^2/m_b^2$

$$f(x) = 1 - 8x + 8x^3 - x^4 + 12x^2 lnx \tag{45}$$

$$f'(x) = (1-x)^4 \tag{46}$$

are the QCD phase space factors and λ_1 and λ_2 correspond to the motion of the heavy quark inside the hadron and the chromomagnetic interaction respectively. These values are taken to be -0.5 GeV for meson and -0.43 GeV for baryon and 0.12 GeV for meson. The chromomagnetic energy is zero for all baryons except Ω_Q.

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Equation (44) is further supplemented by the FQO at the order $(1/m_Q^3)$ in the HQE. At this order due to the light quarks there are three processes: Pauli interference (PI), Weak Annihilation (WI) and Weak Scattering (WS). The PI plays a predictive role in the charged meson and the Λ_b baryon. The PI becomes constructive at the tree level whereas it becomes destructive if radiative corrections are considered. The WS takes place in the neutral meson as well as Λ_b.

1. Lifetime ratio of B^- and B_d

Although the difference between the lifetimes of the charged and the neutral B-mesons is almost a settled issue, we check the once again using the expectation values of the colour-straight operators. This difference is attributed to PI and WA. Neglecting the WA as it is strongly CKM suppressed the result for the PI is

$$\Delta\Gamma^f(B^-) = \Gamma_0 24\pi^2 C_0 \frac{< O_V^q >_{B^-} - < O_A^q >_{B^-}}{m_b^3} \tag{47}$$

where $C_0 = c_+^2 - c_-^2 + \frac{1}{N_c}(c_+^2 + c_-^2)$ and $\Gamma_0 = \frac{G_f^2 m_b^5 |V_{cb}|^2}{192\pi^3}$. The values for Wilson coefficients are: $c_+ = 0.84$ and $c_- = $ -1.42 with $N_c = 3$. For Γ_0, $m_b = 4.8$ GeV and $|V_{cb}| = 0.04$ are used. Then the ratio is

$$\frac{\tau(B^-)}{\tau(B_d)} = 1.00 \tag{48}$$

This agrees well with the one obtained in terms B-meson decay constants.

The decay rates due to spectator quark(s) processes are: For B^-,

$$\Delta\Gamma^{nf}(B^-) = \frac{G_f^2 m_b^2}{12\pi}|V_{cb}|^2(1-x)^2[(2c_+^2 - c_-^2)\phi_1 + 3(c_+^2 + c_-^2)\rho_1] \tag{49}$$

Hence the ratio is 1.03.

2. Lifetime ratio of B^- and B_s

The lifetimes difference between the two neutral mesons B_s and B_d is due to W exchange. The numerical result is

$$\frac{\tau(B_s)}{\tau(B_d)} = 1.00 \tag{50}$$

Corresponding to the nonfactorisable part, we get the decay rate

$$\Delta\Gamma^{nf}(B_s) = \frac{G_f^2 M_b^2}{12\pi}|V_{cb}|^2(1-4x)^{1/2}[\frac{(2c_+ - c_-)^2}{3}((1-x)\phi_1^s - (1+2x)\phi_2^s) + \frac{(c_+ + c_-)^2}{2}((1-x)\rho_1^s - (1+2x)$$

$$\tag{51}$$

Therefore the ratio becomes 1.02.

3. Lifetime ratio of Λ_b and B^-

In the HQE, the difference in lifetimes between mesons and baryons begins to appear at order $1/m_Q^2$. Nevertheless it is dominant at third power in $1/m_Q$. At this order, the FQO receives corrections due to WS and PI. They are

$$\Gamma_{WS}(\Lambda_b) = 92\pi^2\Gamma_0 c_-^2 \frac{< O_V^q >_{\Lambda_b}}{m_b^3} \tag{52}$$

$$\Gamma_{PI}(\Lambda_b) = -48\pi^2\Gamma_0 C_1 \frac{< O_V^q >_{\Lambda_b}}{m_b^3} \tag{53}$$

where $C_1 = -c_+(2c_- - c_+)$. As mentioned earlier PI is destructive for radiative corrections and enhances the decay rate leading to smaller lifetime for Λ_b. The effect of WS, on the other hand, is colour enhanced and its consequence is smaller. Hence,

$$\frac{\tau(\Lambda_b)}{\tau(B^d)} = 0.79 \tag{54}$$

The deacy rate modified by the nonfactorisable piece is given by

$$\Delta\Gamma(\Lambda_b) = \frac{G_f^2 m_b^2}{16\pi}\bar{\lambda}[4(1-x)^2(c_-^2 - c_+^2) - (1-x)^2(1+x)(c_- - c_+)(5c_+ - c_-)] \tag{55}$$

where $\bar{\lambda}$ stands for the term in Eq. (43). Correspondingly, the ratio is

$$\frac{\tau(\Lambda_b)}{\tau(B^-)} = 0.84 \tag{56}$$

In mesonic cases, the nonfactorisable piece gives a little bit higher values. In particular, the ratio of the lifetimes of the baryon and meson is significantly larger.

VI. CONCLUSION

In this paper, we have evaluated the FQO for beauty hadrons. Though the spectator effects are suppressed by powers of $(\Lambda_{QCD}/m_Q)^3$, in the HQE for inclusive decays, they cannot be neglected. We have expressed the four-quark operators in terms of light quark scattering form factor which are in turn related to the harmonic oscillator wave function. The use of the wave function model is to replace the exponential and two pole ansatz used in [10]. Basically both are same. The distinction arises only due to β, the oscillator strength of the model. Interestingly this simple alternative predicts the lifetimes ratio of Λ_b and B closer to the experimental value.

On the other hand, the nonfactorisable part does not have much effect in the case of mesons. But it keeps still away the ratio between B and Λ_b away from the experimental value. As far the B-mesons are concerned, the present study once again affirms the existing predictions. In this case too, there are omissions like SU(2) and SU(3) symmetry breaking. They may play a role but too negligibly.

Finally, we conclude that we have taken one, which is dominant, of the sources of the preasymptotic effects and shown that it predicts the lifetime of the Λ_b close to the experimental figure. As we have not taken into account all possible corrections to the four-quark operators, the present prediction can be considered at least indicative in order to look into the four-quark as well as six-quark operators more seriously. However given the basis provided in [10], the prediction has to be believed. Of course, this prediction can be checked by lattice studies. A refined analysis of b and c flavoured baryon lifetimes will be published elsewhere.

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